Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
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Summary

One of the most prominent concerns when it comes to climate change is increasing amounts of carbon dioxide trapped in the atmosphere. Oceans, plants, and soil naturally sequester a portion of this carbon dioxide, in turn decreasing the amount expelled to the atmosphere. Plentiful forests are therefore vital to natural carbon dioxide storage. Harvested wood products (HWPs) have been essential to the functioning of human society for many years, however, they gradually release the carbon dioxide that was once stored in the living trees from which they were created. To balance the continuing demand for HWPs with potential benefits of appropriate harvesting while maintaining high enough levels of carbon sequestration, we have created the Carbon Sequestration Forest Management Model (CSFM Model).

The age-structured CSFM model informs managers the makeup of their forest over time based on annual conditions and harvesting practices. Here, forest managers who have implemented harvesting and planting rates that are not sustainable will be able to predetermine the effects on the forest. The population size and composition of the forest is then used as input in a carbon sequestration model that predicts the total amount of carbon dioxide stored annually. In order to address wood harvesting needs, we included a harvesting term that accounts for the amount of harvesting done annually and the amount of carbon dioxide stored in the HWPs.

These findings assisted in creating a decision chart for forest managers to follow. Multiple surveys across the American population were consulted for community, religious, environmental, and economic values. Unlike commonly implemented economic strategies for forest management, the decision chart accounts for social and environmental reactions to forest management along with the economics of tree harvesting. The decision chart is intended to be consulted every five years.

The full CSFM Model is accompanied by a forest management plan that can be applied to any forest. A walk through of an example oak forest with the management plan is included.

Overall, the results suggested that forests with slow growing trees should be subject to limited harvesting when the goal is optimal carbon sequestration. Otherwise, forest managers need to focus on harvesting younger trees if harvesting must be done for economic reasons. Fast growing trees can be harvested at higher rates along with proportionate planting rates for maximal carbon sequestration. The CSFM model is a versatile, age-structured carbon sequestration model that predicts changes to both the forest population and carbon sequestration amounts from both trees and HWPs. Forest managers are then provided with a management plan to be applied to their forest in order to logically balance economic and environmental needs.

Keywords: Harvesting; Carbon Sequestering; HWPs; Management Plan

Age Informed Modeling of Carbon Sequestration in Forests

Accompanied by Forest Management Plan and Decision Chart

February 21, 2022

Contents

1 Introduction

Rapidly changing climates have caused natural disasters to increase in frequency and occur in unpredictable patterns. The average temperature of the Earth has risen 1° C in the last 100 years [4]. One cause of this temperature change is an accumulation of greenhouse gasses in the atmosphere which block heat from escaping. Carbon dioxide, CO_2, is the most abundant greenhouse gas produced by humans. During 2019, carbon dioxide accounted for 80% of greenhouse gas emissions in the United States [1]. To alleviate the warming of the atmosphere and thus limit the effects of climate change, carbon dioxide needs to be sequestered in larger quantities.

Carbon sequestering is the storage of carbon dioxide in plants, oceans, soil, and **harvested wood products** (**HWPs**), preventing it from collecting in the atmosphere. Forests and HWPs take up nearly 25% of carbon emissions globally [15]. However, when trees die and decay, they begin to release their carbon dioxide. The alarming rate at which forests are being harvested for products decreases total global carbon sequestering drastically.

Forest managers are in charge of determining how much of the forest they oversee gets harvested and replanted every year. While many managers sign multi-year contracts for a forest, we created a model that managers can review every five years to make changes to harvesting and planting rates accordingly with the goal of balancing an increased amount of carbon sequestration with social and economic values.

We created an age-structured model that predicts the generational population sizes of seedling, sapling, pole, and mature trees. The generational classes are separated by the United States Forest Service using data about tree diameter and age [13]. A derivative of the Ash and Herman equations [2] were then used to return the total weight of carbon dioxide stored by the forest. We modified this model to account for sequestered carbon in HWPs.

We further analyzed these mechanisms to create a model that optimizes carbon sequestering in a forest while still allowing for economic gains received by harvesting. To help forest managers in their decisions of which management plan to follow we created a decision chart for managers to assist in this reevaluation.

2 The CSFM Model

2.1 Tree Generations Using an Age Structured Model

Carbon sequestering in forests is closely related to the size, species and amount of trees present in the forest. Each forest has an fluctuating number of each tree variety that grows and diminishes with time. The first step in modeling carbon sequestering of a forest over time is to create a model that simulates the population size of each species of tree in the forest. An age-structured model was created, seen in Figure 1.

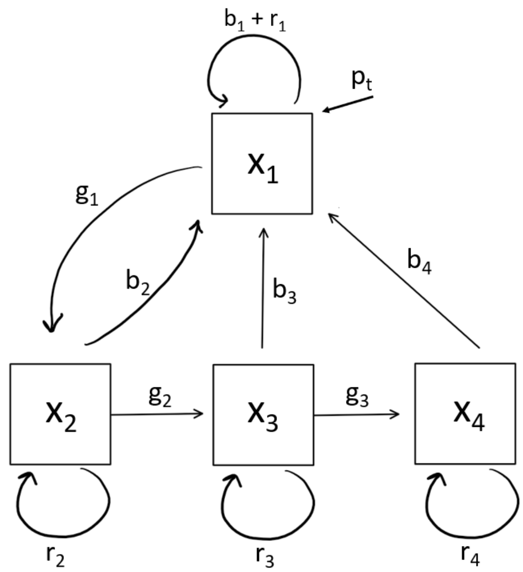


Figure 1: Age-structured Model of Tree Generations

Here,

- x_1, x_2, x_3, and x_4 represent number of trees in the seedling, sapling, pole, and mature stages respectively, as determined by the United States Forest Service [13], assumed to be nonnegative,
- b_i is the **birth** or **reproduction rate**, $0 \leq b_i \leq 1$, of class x_i over one year,
- p_t, the **production** or **planting rate**, $p_t \geq 0$, is the number of trees planted by outside sources per year on year t,
- g_i, the **growth rate**, $0 \leq g_i \leq 1$, represents the proportion of trees moving from class x_i to class x_{i+1} due to diameter growth over one year,
- and r_i, the **retention rate**, $0 \leq r_i \leq 1$, represents the proportion of trees remaining in the class x_i after one year.

Findings by Roxburg et. al. [12] suggest there exist distinct classes for carbon sequestering ability in tree populations. The distinction between classes is directly related to diameter size which can be seen in the research completed by Ash and Herman [2]. We assume the average diameter of the trees in the age classes, $x_1, \ldots, x_4$, according to the United States Forest Service [13] are accurate representations of the trees in each class for ease of understanding carbon sequestration. We further assume both g_i and r_i are constant. This means that as harvesting of trees increases neither g_i and r_i decrease. Harvesting is further accounted for later in the paper. It is important to note that $1 - (g_i + r_i) = d_i$ where d_i is the proportion of x_i that dies each year. The dead trees may have been harvested or died naturally.

Further, assume that there is a set amount of time for trees in each class to proceed to the next. We denote this tm_i, the time to maturity from class i to $i + 1$. Thus we can define a relationship between g_i, r_i, d_i, and tm_i:

$$g_i = \frac{1 - d_i}{tm_i} \quad r_i = \frac{(1 - d_i)(tm_i - 1)}{tm_i},$$

so that $g_i + r_i = 1 - d_i$.

The evolution of class size over time is studied by investigating the iteration of Figure 1 and is described by

$$\begin{aligned} x_1(t+1) &= (b_1 + r_1)x_1(t) + b_2x_2(t) + b_3x_3(t) + b_4x_4(t) + p_{t+1} \\ x_2(t+1) &= g_1x_1(t) + r_2x_2(t) \\ x_3(t+1) &= g_2x_2(t) + r_3x_3(t) \\ x_4(t+1) &= g_3x_3(t) + r_4x_4(t). \end{aligned} \tag{1}$$

Iterating Equation 1 over multiple years returns the total number of one species in each class every year per year. This process is then repeated for the other species in the forest. Once the population size for each class is determined, we then approximate the carbon sequestered by each species.

2.2 Approximating Carbon Sequestered

Given class population sizes x_1, x_2, x_3, x_4 at a specific year, we wish to be able to approximate the total amount of carbon sequestered. As is modeled by [12], we can approximate the amount of carbon sequestered by a single tree using its **diameter at breast height** (**D.B.H.**), assuming that carbon is 50% of tree biomass. Figure 1 of [12] provides a graph of carbon sequestration of a tree, in tonnes of carbon (tC), against its D.B.H. They provide an explicit quadratic formula for carbon sequestration given diameters $D > 145$, however using the Ash Helman formula along with their formula for adjusted biomass, we acquired results that did not align with their Figure 1.

Instead, we modified the given equations in order for the carbon function to maintain continuity and to replicate their figure. We did this by keeping their quadratic formula for $D > 145$, and then generating an exponential growth formula $e^{\xi D} - 1$ on $0 \leq D \leq 145$, which is zero at $D = 0$ and $\xi \approx 0.01548$ is the exact real value to maintain continuity at $D = 145$.

$$C(D) = \begin{cases} e^{\xi D} - 1 & 0 \leq D \leq 145 \\ -0.0004D^2 + 0.234D - 17.077 & 145 < D \end{cases} \tag{2}$$

We note that Formula 2 replicates a curve where the input D is in centimeters, and the output carbon amount is in tonnes carbon. Therefore, given populations x_1, x_2, x_3, x_4 which we assume to have average D.B.H.s D_1, D_2, D_3, D_4 respectively, we approximate the total carbon sequestered by multiplying the carbon sequestered per tree by the total number of trees in that class

$$\sum_{i=1}^{4} x_i C(D_i).$$

This summation assumes that using an average D.B.H. value for an age class in calculation results in an accurate approximation, rather than summing for individual trees' diameters. Moreover, the calculation of carbon sequestration per tree is independent of the density of the population it is in. Using this method, we can calculate the total amount of carbon sequestered by a forest per year, and evaluate how this value changes from year to year. For modeling several species of tree in a forest, we would assume independence by repeating this process and summing together the total carbon sequestered by each species.

2.3 Accounting for Harvesting

We accounted for tree harvesting by removing some amount from each class annually. At time t, we subtract $h_{i,t}$ from $x_i(t)$. Consequently, the amount of carbon sequestered from harvested wood products (HWPs) evolves according to the following equation [8]:

$$\phi_{t+1}^j = e^{-k}\phi_t^j + \left[\frac{1-e^{-k}}{k}\right]\left(\sum_{i=1}^{4} h_{i,t+1}C(D_i)\right)\alpha^j\eta^j, \tag{3}$$

in which ϕ_t^j is the carbon stored in HWP j at time t. The decay parameter $k = \frac{\ln 2}{HL^j}$, where HL^j is the half life of the respective HWP. The number of harvested trees from a particular class at time t, $h_{i,t}$, is allocated to HWP j based on some proportion $0 \le \alpha^j \le 1$. We assume that the entirety of the biomass of the tree is used for HWPs so that $\sum_j \alpha^j = 1$. The carbon factor, η^j, tells us how much carbon a single HWP sequesters when it is first produced to account for the carbon lost in production. We will assume that each HWP falls into one of three categories (Table 1).

HWP j			
	Sawnwood	Wood-based panels	Paper & Paperboard
Half-life (years); HL^j	35	25	2
Carbon factor (tC m^{-3}); η^j	0.229	0.269	0.386

Table 1: Half-life and carbon factor for three types of harvested wood products [8].

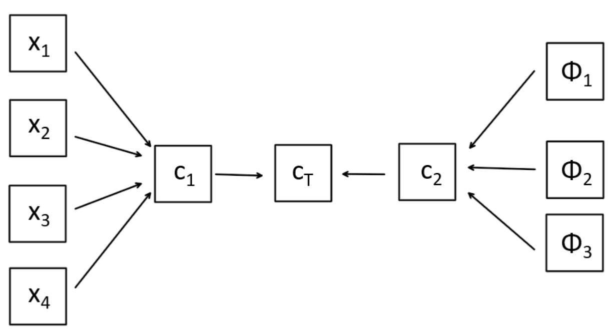


Figure 2: Total carbon sequestration (c_T) as determined by living trees (x_i) and harvested wood products (ϕ_i).

Through accounting for harvesting, we modify Equation 1 to obtain the CSFM equations for forest population:

$$
\begin{aligned}
x_1(t+1) &= (b_1 + r_1)x_1(t) + b_2 x_2(t) + b_3 x_3(t) + b_4 x_4(t) + p_{t+1} - h_{1,t+1} \\
x_2(t+1) &= g_1 x_2(t) + r_2 x_2(t) - h_{2,t+1} \\
x_3(t+1) &= g_2 x_3(t) + r_3 x_3(t) - h_{3,t+1} \\
x_4(t+1) &= g_3 x_4(t) + r_4 x_4(t) - h_{4,t+1}.
\end{aligned} \tag{4}
$$

In addition, the CSFM model counts total amount of carbon sequestered at time t through the sum of the carbon held in the trees and that stored in HWPs:

$$\sum_j \phi_t^j + \sum_i x_i(t) C(D_i), \tag{5}$$

shown visually in Figure 2.

3 Application of CSFM

3.1 Equation Manipulation for Application for Sequestration

In applying our model to a living forest, we wish to be able to analyze how much harvesting and planting can be done in order to maintain the forest population. Equation 4 can be rewritten as the following matrix equation:

$$\vec{x}(t+1) = M\vec{x}_t - \vec{h}_{t+1} + \vec{p}_{t+1} \tag{6}$$

$$M = \begin{bmatrix} r_1 + b_1 & b_2 & b_3 & b_4 \\ g_1 & r_2 & 0 & 0 \\ 0 & g_2 & r_3 & 0 \\ 0 & 0 & g_3 & w_4 \end{bmatrix}, \quad \vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}, \quad \vec{h}_t = \begin{bmatrix} h_{1,t} \\ h_{2,t} \\ h_{3,t} \\ h_{4,t} \end{bmatrix}, \quad \vec{p}_t = \begin{bmatrix} p_t \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Moving forward, we will set $b_1, b_2 = 0$ as trees typically will not be sexually mature until later in their lives [7]. We further assume that the forest manager can contract the maximum amount of harvesting allowed per year. Therefore, given an initial population of a forest, our model should be able to suggest an amount of harvesting and planting to be done in accordance with the desired outcome. One wish that a forest manager may have is to maintain the size of the forest while planting and harvesting in amounts that offset each other. To find this, we solve our dynamical system for fixed points at an arbitrary time.

$$
\begin{aligned}
\vec{x} &= M\vec{x} - \vec{h} + \vec{p} \\
\implies \vec{p} &= \vec{h} - (M - I)\vec{p}.
\end{aligned}
$$

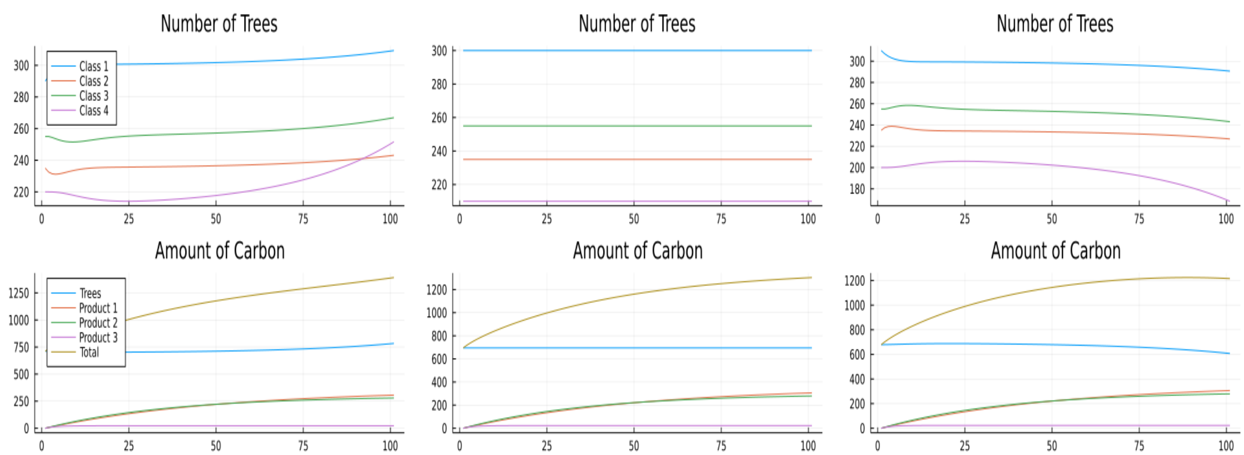
Solving component wise,

$$
\begin{aligned}
p &= h_1 - (r_1 - 1)x_1 - b_3 x_3 + b_4 x_4, \\
h_i &= g_{i-1} x_{i-1} + (r_i - 1)x_i \quad \text{for} \quad i = 2, 3, 4.
\end{aligned} \tag{7}
$$

Equation 7 tells us that with fixed parameter values for the growth, retention, and birth rates, and given an initial population $\vec{x}_0$, we can directly solve for steady state values h_2, h_3, h_4. Also, each additional tree of the youngest class that is harvested results in one additional tree to plant. Moreover, harvesting the seedlings does not make much financial sense. Therefore we set $h_1 = 0$ yielding an exact steady state solution for p. For forests with older populations or low retention rates, such a solution may yield negative numbers. For the case when a positive solution exists, harvesting and planting at the amounts provided results in maintaining a steady annual population. This is helpful as harvesting still occurs resulting in increased total sequestration, but the population of the forest does not vary drastically. In addition, this equilibrium solution can help a forest manager understand how deviations from this amount would result in changes to the forest population.

3.2 Carbon Optimization

As is shown in Figure 3b, the CSFM steady state approach involves planting and harvesting at the rates obtained from Equation 7. This results in constant population size and composition while steadily increasing carbon sequestration from amounts stored in the HWPs. However, Figures 3a and 3c display the potential sensitivity to perturbations of the initial population over a 100 year period. A slight change could result in eventual decay or exponential growth to the population. This management approach is beneficial in that it supports the HWP economy through harvesting, which in turn sequesters more carbon while not overpopulating the forest. However, this plan is limited in that we may not get realistic harvesting values for certain forest compositions. Some initial forest conditions return negative harvesting values, due to the fact that it is not always possible to maintain the exact amount of trees in each class. This is especially the case for older forests with a majority of trees in classes three and four, as planted trees must enter classes one and two before they can replenish the upper classes. A solution to this is addressed in the forest management plan.



(a) (b) (c)

Figure 3: Near steady state approach. The number of white oak trees (Table 2) and amount of carbon sequestered through trees and HWPs over a time span of 100 years. Product 1: Sawnwood, $\alpha = 3/8$; Product 2: Wood-based panels, $\alpha = 3/8$; Product 3: Paper & Paperboard, $\alpha = 1/4$. Trees are planted at a rate of 52.65 trees per year. Classes are harvested at amounts of about 0, 15.25, 23.86, and 32.80 respectively. The initial populations are (a) $[290, 235, 255, 220]$, (b) $[300, 235, 255, 210]$, and (c) $[310, 235, 255, 200]$, showing sensitivity to initial conditions.

Accounting for the fact that forest managers will want to reevaluate their management plan on a regular basis, we include a cycling harvesting approach. This alternates between a set harvesting amount for five years, and no harvesting the following five years (Figure 4). This plan resulted in total carbon amounts that are slightly higher than in the steady state solution; this could be due to the population shifting slightly towards older and larger trees that can sequester more carbon annually. Forest managers would have to be careful as to not let the forest size vary drastically between these five year increments. Moreover, this may not align with the economic needs of the community, as the demand for HWPs may be high in the years of no harvesting. This is where it is critical for the forest manager to consult the decision chart rather than simply relying on the mathematical model. If the forest manager chooses to reevaluate every five years, they will modify the harvesting and planting rates as need be. Figure 4 merely shows one mean for this reevaluation to occur.

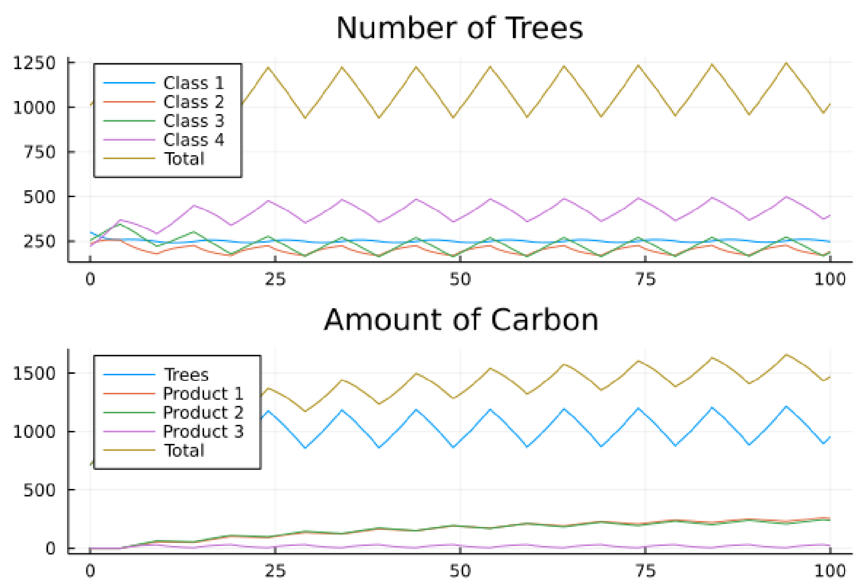
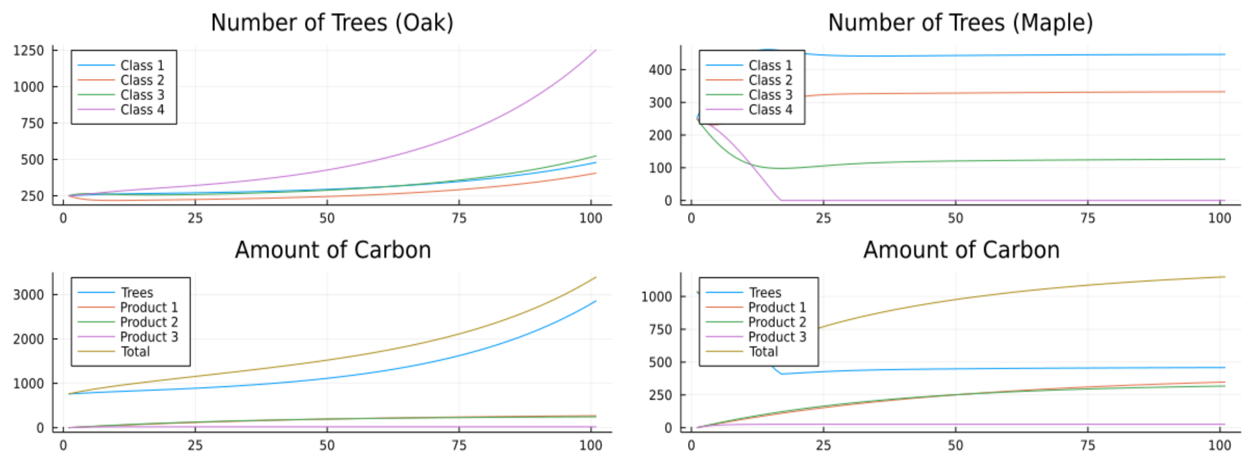


Figure 4: Cycling harvesting approach: The number of white oak trees and amount carbon sequestered through trees and HWPs over 100 years. Compared to parameters in Figure 3, product amounts are the same, planting rate is 60% of previous, harvesting amounts are multiplied by 1.633, and initial populations are $[300, 235, 255, 210]$. Harvesting occurs for five years, followed by five years of only planting.

3.3 Comparison of Forests

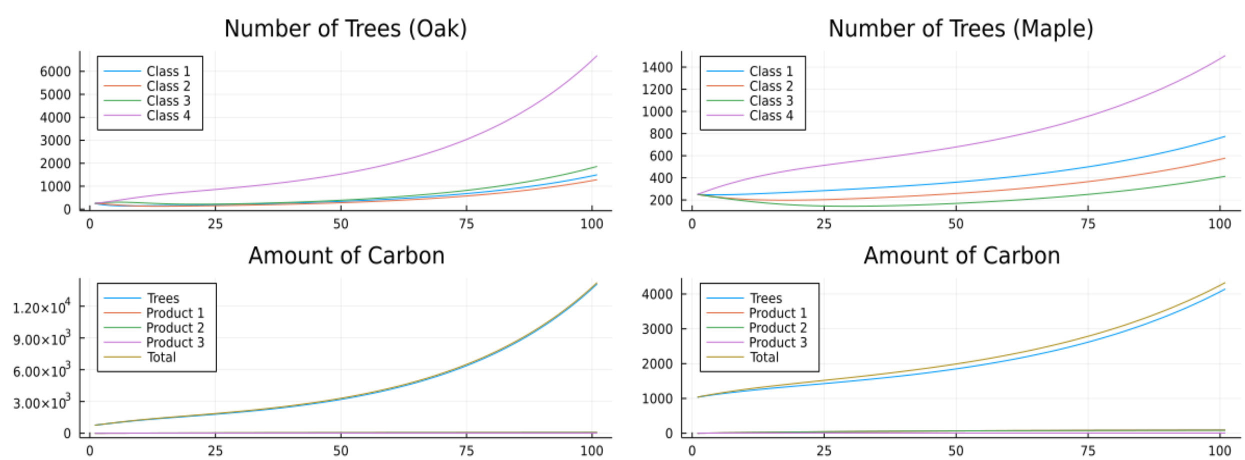
From collected data on two species of trees [13] (see Table 2), we can model the population and carbon sequestration of forests containing white oak and sugar maple trees. Since our model assumes independence between species of trees, we will imagine forests that are either fully white oak or sugar maple. White oak trees mature quicker than sugar maple trees and in general have a lower D.B.H. In this section, a forest will contain 1000 trees of one species, with some distribution of those being in each age class.



(a) (b)

Figure 5: Comparison of initial population $[250, 250, 250, 250]$, 40 trees planted annually, $\vec{H} = [0, 10, 20, 30]$, $\vec{\alpha} = [3/8, 3/8, 1/4]$ between (a) white oak and (b) sugar maple over 100 years.

As can be seen in Figure 5, the same harvesting and planting amounts in forests of the different species can result in vastly different outcomes over 100 years. Oak trees were able to reproduce and generate exponential growth, while the constant harvesting of the oldest maple trees exceeded the rate at which they were replenished. The oak tree forest therefore had higher carbon sequestration than the maple tree forest, which seemed to have hit an equilibrium around a similar level to the initial amount.



(a) (b)

Figure 6: Comparison of initial population $[250, 250, 250, 250]$, 5 trees planted annually, $\vec{H} = [6, 6, 6, 6]$, $\vec{\alpha} = [3/8, 3/8, 1/4]$ between (a) white oak and (b) sugar maple over 100 years.

Figure 6 displays simulations where both forests see exponential growth over time. The rate of aging seems to be the most influential factor towards how much harvesting can be done. Cutting down older trees and allowing for younger trees to grow tends to show optimal results for the oak trees that can quickly replenish the oldest class. On the contrary, cutting down too many older sugar maple trees causes extinction of that class, requiring a shift in harvest strategy. As an example, when the same procedure as in Figure 4 was repeated on sugar maples, the trees died out. The amount of harvesting was required to be reduced to match the graph.

This comparison of the white oak and sugar maple trees demonstrates that knowing the time it takes for trees to age is essential if planning to harvest by D.B.H. For trees that grow much quicker, more aggressive harvesting approaches can be used on the older populations as the younger populations will in less time replenish that group. In considering forests with multiple types of trees, knowledge of their aging and D.B.H. values will help determine how best to sequester carbon from the atmosphere.

3.4 Decision Model

The management plan proposed for optimization of total carbon sequestering while allowing for harvesting presents a model efficient at prioritizing environmental action. However, there are a variety of reasons in which forests should not or cannot be harvested to the suggested amounts. Due to societal, economic, religious, and environmental concerns, a decision chart has been created to assist forest managers in their plan. According to Judith Stein, a member of MIT's human resources, consensus is important in making successful decisions [14]. Our decision chart, visually represented for ease of use in Figure 7, is based on consensus information taken from surveys of the community surrounding the forests [6] [9] [10].

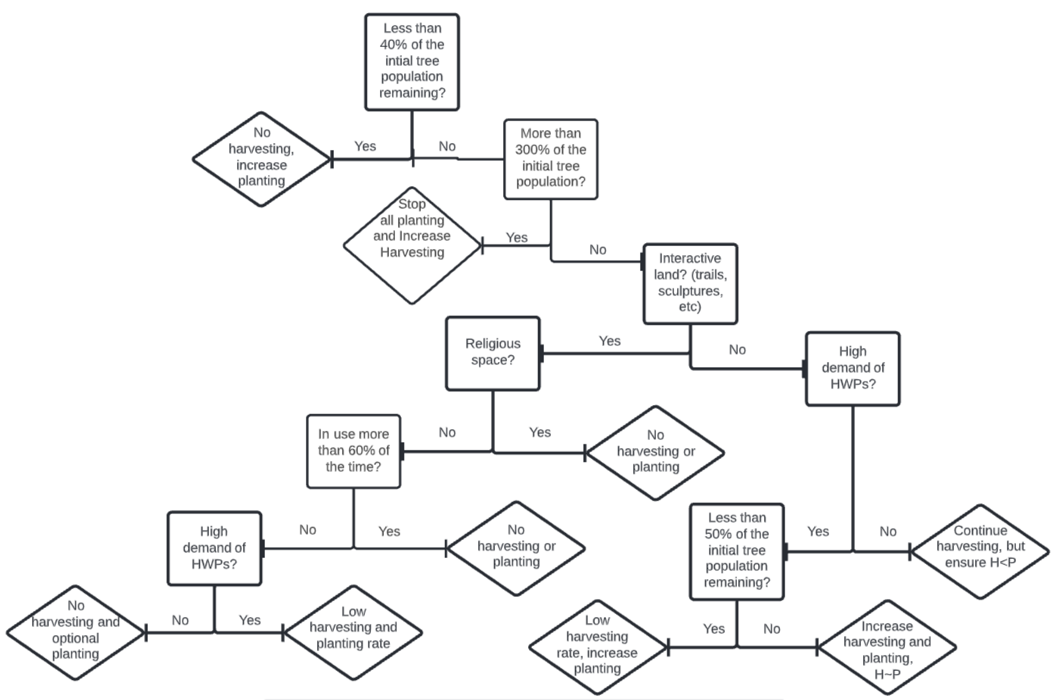


Figure 7: Decision model to determine harvest rate (H) and planting rate (P).

The largest remaining rainforest in the world is the Amazon rainforest which spans four countries. There are now laws in effect that protect 40% of the rainforest from being clear cut [3]. We assume that this international protection threshold of 40% can be viewed as community consensus. Thus, the first criteria in the decision chart asks if the forest in question is below 40% of it's initial size. If it is below the threshold, the forest manager must immediately stop any harvesting and only plant trees for the next five years. Importantly, the total carbon sequestered would be greatly minimized if the amount of trees were allowed to decline much further.

If the forest population is above 40% of its initial size, the forest manager then ensures that the forest is below 300% of its original population. For a healthy forest, we make the assumption that tripling the original population would begin to negatively impact the ecosystem. Thus if a threshold of 300% is hit, planting of new trees must stop and harvesting should increase to provide necessary space.

If the number of trees is below this threshold, then the forest manager must determine if the land can be considered an interactive space. We define an interactive space as one that is used by community members to spend time or gather. Examples include: trails, camping, artwork, fairs, and other gathering events. If the space is interactive, the forest manger must determine whether the space is used for religious reasons. If this is the case, the land should neither be harvested nor planted on. A survey of Americans in 2020 [9] showed that there is a strong consensus that religious spaces are not to be interrupted. If the interactive land is not used as a religious space, the forest manager must determine if the land is in high use ($\geq$ 60% of the time). If so, no harvesting or planting should occur because of the effect it will have on the community. If the land is not in high use and demand is high for HWPs, then harvest and replenish accordingly. This will allow for a total carbon sequestration balance by creating new trees that will grow and creating products that store carbon. If HWP demand is low, there is no need to harvest. Here, planting to create a larger stock of trees is encouraged for future economic benefit.

If the land is not interactive, then supply and demand of HWPs is an important factor for the forest managers decision. The United States Forest Service states, "globally the forest products industry contributed to over 1.1 % of gross domestic product (GDP) and 1.2 % in total employment opportunities to the global economy in 2014," [6]. If there is high demand for HWPs, then harvesting and planting can continue contingent upon the tree population being above 50% its original size. If the size is below 50% and demand is high then a low harvesting rate and a higher planting rate can be implemented. This may have negative economic impact, but a survey from 2020 reported that 63-69% of Americans believe that higher costs were worth environmental action strides [10]. A threshold of 50% was chosen because it is within 10% of the lower bound for forest size and it is possible for 10% of forest size to decrease in one five year time period. If demand for HWPs is low, then harvesting may continue as long as planting occurs.

As the harvesting and planting rates are assumed to be reevaluated every five years, we suggest consulting this decision chart accordingly. This decision chart accounts for social, economic, carbon sequestering, and religious concerns of communities surrounding the forests.

As the forest manager follows the decision chart, the management plans may need to be adjusted. We expect this transition to occur in five year increments for all forests. There are certain instances when a transitional plan will be required, regardless of the forest in question. Our simulations show that older tree populations tend to increase faster than the younger classes. Thus, when the old tree population in a forest becomes too large, attention needs to be focused on harvesting the old trees and planting new ones that will ultimately replenish the forest.

As may be expected, if the planting rate is too much greater than the harvesting rate, the forest will grow exponentially. Likewise, too great a harvesting rate will result in the a drastic decrease in tree population over time. The potential occurrence of either of these cases presents a clear problem. It is therefore vital to reevaluate the management plan every five years. The relatively bounded forest growth observed in the cycling approach in Figure 4 shows that the forest can be controlled if harvesting and planting rates are adjusted accordingly.

Each forest will also have its own set of factors that present the need for a transitional plan. Every community will have its own values surrounding forest use, again stressing the importance of consensus in decision making. Moreover, forest composition will vary by location, both in terms of species and class distribution. This will lead to differing growth, death, and reproduction rates for all forests, so some forests may need to shift managements plans more often than others.

Depending on location, there are concerns that will only be applicable to certain forests. For instance, a dry area that is prone to drought and wildfire will likely have lower growth rates and higher death rates of trees, so reevaluation may have to occur more often depending on the state of the forest. There will be instances in which a transition to the management plan must take place, even if it occurs outside of the regular five year evaluation cycle. This is especially true for a location with irregular weather patterns or the occurrence of a natural disaster.

4 Walk-Through Forest Management Plan

4.1 Generic Management Plan

Next we incorporate the information from both the steady state approach and the decision chart to make a generic management plan for any forest. To begin this plan, find the initial population size of each tree species in the forest, an approximate distribution of trees by their age class, and the corresponding parameters for the trees.

1. Find the steady state of each tree species by solving Equation 7 with the assumption $h_1 = 0$. This will return the harvesting rates for each class of the tree species as well as the planting rate required to maintain steady state population.

2. If any harvesting rates are negative, set the negative proposed rates to zero and increase the suggested planting rate in a near one-to-one fashion. Similarly, if the proposed planting amount is negative, set that to zero and increase the proposed harvesting amounts.

3. The forest manager considers the proposed rates and then adjusts them to meet required harvesting amounts or maximal planting ability.

4. If the forest manager wishes to grow or reduce the forest population, they then adjust the suggested planting and harvesting amounts accordingly.

5. Consult the decision chart every five years to modify the harvesting and planting amounts.

4.2 Oak Forest Management Plan

We will walk through the management plan with an example oak forest. The simulations on oak forests show that these trees grow relatively fast, making them a good candidate for harvesting.

Given an initial population of $\vec{x}_0$ = [300,235,255,210] white oak trees as used in Figures 3 and 4, we provide example walk-throughs for two hypothetical forest managers. The first forest manager wishes to maintain the population of the forest around 1000, has the supplies to plant 40 trees per year, and would like to harvest as much as they can given the harvesting amount. At year zero, the model proposes a steady state of $\vec{H} \approx$ [0,15,24,33] and $P \approx 53$. Since the forest manager can only plant 40 trees annually, we propose $\vec{H}'$ = [0,15,24,20] and P' = 40, where thirteen less old trees are harvested to account for planting thirteen less then proposed. So for the first five years, 59 trees are harvested and 40 trees are planted annually. After five years, the new tree population is $\vec{x}(5) \approx$ [266,219,250,273] for a total of about 1007 trees. At this time, the model proposes a steady state of $\vec{H} \approx$ [0,11,21,32] and $P \approx 41$. Again, a slight shift is made such that $\vec{H}'$ = [0,11,20,31] and P = 40, resulting in a new tree population $\vec{x}(10) \approx$ [263,217,251,278] for a total of about 1009. Continuing this process gives us a final population $\vec{x}(25) \approx$ [263,219,254,274] for a total of 1010 trees in the forest. The iterated propositions seem to have converged on the desired number of trees planted annually as the tree population shifted, and we note that this process could likely continue as long as the forest manager has the desired planting rate of 40. See Figure 8 for how total carbon sequestration amounts increased over the twenty-five years simulated.

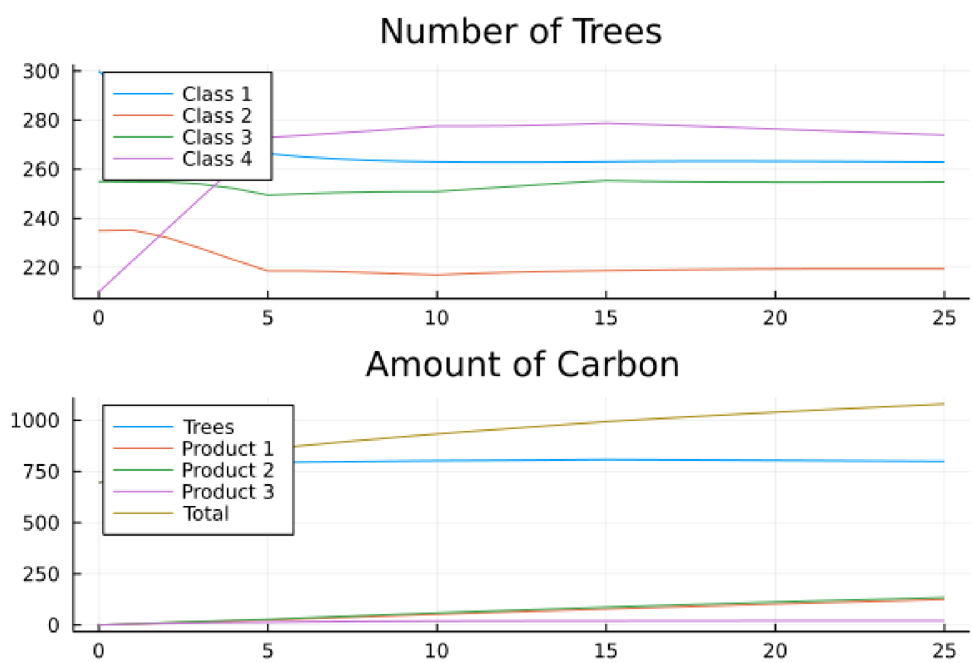


Figure 8: Oak Tree simulation with five-year adjustments with initial population [300,235,255,210], a desire to plant 40 trees annually, and α set as in Figure 3.

The second forest manager also wishes to maintain the 1000 oak trees, however demands that 100 trees are harvested annually for forest products. Similar to before, given the initial proposed steady state $\vec{H} \approx$ [0,15,24,33] and $P \approx 53$, we make an adjustment such that the sum of $\vec{H}'$ is 100 trees, proposing instead $\vec{H}'$ = [0,24,33,43] and P' = 81, where we uniformly add amounts to each harvesting group besides the youngest class, and then add roughly the same amount to P. Figure 9 shows the results of this simulation, and again it can be seen that after roughly ten to fifteen years, a near steady state is achieved.

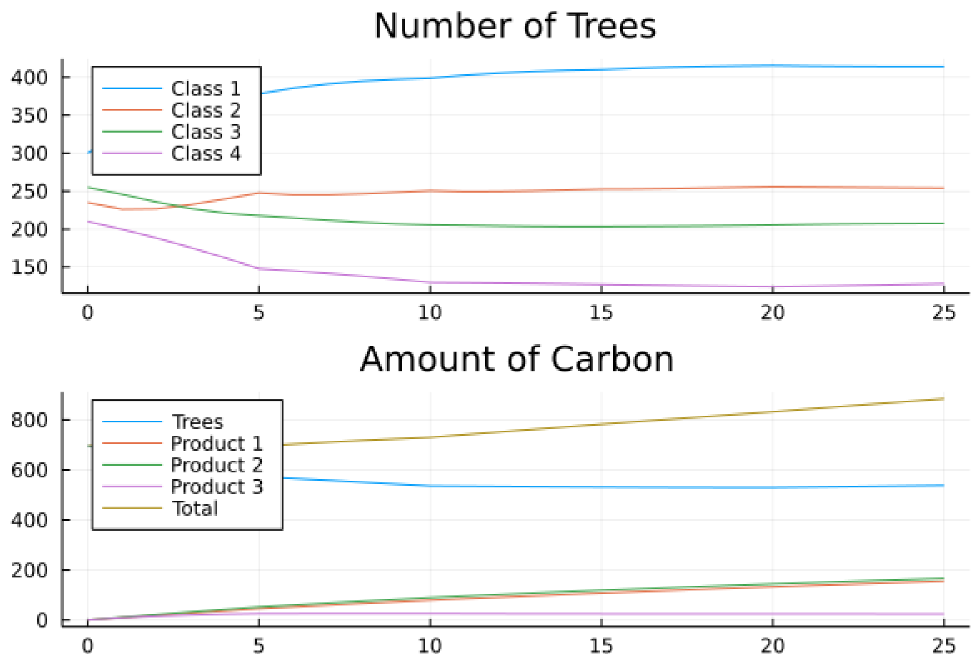


Figure 9: Oak Tree simulation with five-year adjustments with initial population $[300, 235, 255, 210]$, a desire to harvest 100 trees annually, and α set as in Figure 3.

5 Discussion and Conclusion

The CSFM model is an age-structured model that groups trees into four distinct age classes based on D.B.H. These classes are seed, sapling, pole, and maturity. Using relative rates of birth, retention, growth, and planting, we model the populations of the four classes of trees per year time. We then use a carbon sequestering model that predicts the amount of carbon sequestered by each of those trees annually. Finally, we use a harvesting model to account for the amount of trees that are cut annually and converted to HWPs, and how much carbon those HWPs store. Using this model, and data from Table 2, we modeled the tree populations and resulting carbon sequestration amounts.

Before analysing the model, it is important to note four combinations of planting and harvesting rates. If planting is high and harvesting is low, then the forest will grow exponentially, leading to overpopulation. A high harvesting rate and low planting rate will ultimately cause the forest to die off completely. Interesting dynamics occur when planting and harvesting rates are relatively equivalent. These dynamics are studied furthered in the simulations presented.

Each year, wood from forests is cut down to create various products for consumer use. With the knowledge that there will be continued demand for HWPs, and that the companies that cut down wood will likely have a contract with an amount of wood they can cut annually, the CSFM model assumes a number of trees to be cut down annually for forest products. When a tree is harvested and a forest product is made, a significant amount of its carbon is released into the atmosphere and the remaining carbon goes into that product. Onwards, carbon is released from that product in an exponential manner [8]. Modeling this takes into account the carbon that is still contained in these products, and reveals to us the strategy that a steady state solution where harvesting is non-zero results in increased total carbon sequestration. However, through simulation shown in Figures 3 and 4, since a large amount of carbon from the trees is released when products are made, the most beneficial plan for maximizing carbon sequestration involves maintaining or growing the forest population.

Through simulation, and as can be seen in Figures 5 and 6, we can see that trees with different parameters related to their age classes, growth rates, and death rates influence the dynamics of tree population and carbon sequestration. In particular, it seems that with the faster growing tree, the white oak, higher amounts of harvesting still allow for steady or exponential increasing solutions than in the slower aging sugar maple. We hypothesize that it is less efficient to harvest slower aging trees as it then takes longer for newly planted trees to replenish those that are cut down. This suggests it is more beneficial to harvest younger slow-growing trees so that the old trees may continue to reproduce. More so, it suggests that when looking for best carbon sequestration and economic benefits of harvesting, slow growing tree management plans are less likely to include harvest.

We have identified two primary management approaches which each have their drawbacks and limitations. The first plan, the steady state approach represented in Figure 3, assumes that the forest is currently at a healthy population that is wished to be maintained. Solving Equation 7 and modifying the results as needed provides harvesting and planting rates that relatively maintain the forest population and population distribution over the next five years. This plan increases the amount of carbon sequestered over time through HWPs, and is beneficial to the economy through its constant harvesting. Deviating from this plan could result in large growth or decay of the forest population. With the information of the steady state harvesting and planting rates, were the forest manager wishing to increase or decrease the forest population, they could deviate from the desired steady state.

A second approach that has been found to sequester equal or higher amounts of carbon is the cycling harvesting approach (Figure 4). This approach allows for larger amounts of harvesting for a set amount of time, followed by a set amount of time of only planting. Depending on the economy and HWP demands, this approach may not always line up with the needs of the community. This could be overcome by having two nearby forests on opposite five year cycles so that carbon sequestration is optimized and harvesting demands are consistently met.

The management plans are expected to be reviewed every five years. During these reviews, the forest manager will assess environmental, social, and economic values for the current year. The management plan will then be modified to fit current events. In Section 4.2, we iterated two examples of this plan utilizing the steady state approach. In both cases, we saw an increase in carbon sequestration while maintaining near steady state population. This model can suggest to the forest manager the number of trees of various sizes to harvest and the number of new trees to plant each year in order to relatively maintain the same overall population. If the goal of the forest manager is not to maintain population, they can modify harvesting or planting amounts for the desired result. With the goal of increasing the population, the forest manager can increase planting or decrease harvesting, and visa versa for decreasing the population. They can later reevaluate to ensure that carbon sequestration and forest population goals are on-track. Interestingly, in both oak example management plans, the populations of each age class seemed to find a near steady state after ten or fifteen years. This suggests that the model is flexible to different requirements and will allow for forest managers to find a steady state plan in a relatively short amount of time given reasonable requirements.

Our model is limited in that the initial forest composition may not return a valid steady state solution. For instance, it often returns negative harvesting values, which is not realistic. However, our simulations show that this can be overcome by replacing a negative harvesting value with an increased planting rate in a one-to-one ratio. Similarly, decreasing harvesting amounts while increasing the planting amount in a one-to-one ratio seems to also result in near steady state solutions. However, our model can be sensitive to initial conditions. As we saw in Figure 3, subtracting ten trees from the initial population of saplings while increasing that of the mature trees resulted in eventual exponential growth, while doing the opposite action resulted in a decreasing population of trees. Importantly, these changes took several years to take place, so using a five year reevaluation plan should be able to correct for such an issue. Additionally, the age-structured model assumes a mean D.B.H. for each class. This may not be representative of the actual forest population, as the actual D.B.H. could be skewed from the mean. Likewise, assuming unchanging birth and death rates could be a problem. These rates will change through the seasons. As the climate changes, the tree parameters will have to be adjusted accordingly.

There are many ways we could further explore and modify the CSFM model given more time and resources. So far, we have only applied our model to forests of a single tree species with varying distributions of age classes. Due to a lack of data, our model has not been validated against the population growth of a real forest or the exact amounts of carbon sequestered per tree of a given D.B.H. Therefore, this model may require reparameterization after it is run on real forests and true population data is collected. For example, the birth and death rate of trees in a given forest may vary from the approximated values obtained from [5] or [11]. In addition, we would run the model with multiple tree compositions to more accurately simulate the population of a diverse forest. Along the same lines, running our model in different climates would encompass the variety of forests that exist around the world. Our current plan that maintains steady state tree composition ensures that the class distribution within the population remains the same.

Another step we would like to take is to solve for steady states more generally from Equation 7 by requiring that the sum of $\vec{x}_{t+1}$ is equal to the sum of $\vec{x}_t$. This would result in many solutions that all maintain a steady total population, rather than just maintaining the exact populations in each age class. This could allow for a forest with many young trees to maintain total population yet mature to a forest with older larger trees that can sequester more carbon. Additionally, we would explore how changing the planting rate over time would affect our results, similar to what we did with the cycling harvesting approach. Finally, further investigation of forest management and the HWP economy would better inform our model. We would look into how the cost of planting and harvesting compares to the economic gain of HWPs.

6 Strengths and Weaknesses

6.1 Strengths

- **Versatility**
  This model can be applied to any forest as long as the parameters are known for birth, death, growth, and retention rates.

- **Steady forest population**
  Both the steady state and cyclic harvesting approach maintain a relatively steady forest population.

- **Ability to reevaluate**
  The inclusion of a decision model allows the forest managers and the community to reevaluate harvesting and planting rates based on the needs of the forest and the community.

6.2 Weaknesses

- **Sensitivity to initial conditions**
  Slight deviations in harvesting and planting rates can lead to drastic long term growth or decay.

- **Assumes constant birth and death rates**
  Birth and death rates may change based on location, climate, and seasonal patterns.

- **A steady state solution may not exist for all forests**
  Initial forest compositions, especially those with larger trees, may return negative harvesting values required to maintain steady state.

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Appendices

Appendix A Tree Information

Classifier	White Oak	Sugar Maple
D.B.H. Class 1 (mean) (cm)	5-14 (9.5)	6-20 (13.0)
Time Class 1 to 2 (Yrs)	4.0	10.0
D.B.H. Class 2 (mean) (cm)	14-29 (19.5)	25-36 (30.5)
Time Class 2 to 3 (Yrs)	4.0	10.0
D.B.H. Class 3 (mean) (cm)	29-42 (35.5)	46 - 59 (52.5)
Time Class 3 to 4 (Yrs)	7.55	10.0
D.B.H. Class 4 (mean) (cm)	42-91 (66.5)	53.6-91 (72.3)
Death Rate	3-12% (7.5%)	0.4%

Table 2: Model required data by tree species [11] [5]

Dear Community,

Today we reach out to you on behalf of the forest managers near your community to discuss tree harvesting. Tree harvesting can be perceived as a destructive and environmentally costly practice. It is well known that trees hold carbon dioxide which, when released, accumulates in the atmosphere contributing to climate change. Tree harvesting can also lead to barren land which is disconcerting for a community. However, there is a high demand for timber every year. This leaves us with a difficult reality: wood products are a necessity for everyday life, both in terms of production and employment. In 2014 the forest harvesting industry provided 1.2% of global employment opportunities. So we must answer an important question: How does a society harvest trees in an economically, socially, and environmentally friendly way?

This is where we can offer you an answer. Our team has developed a harvesting model that can balance the economic demand of harvesting with the environmental impacts of carbon dioxide storage within trees. We suggest that the forest managers near you follow a half-decade reevaluation method. This method requires forest managers to check the harvesting and carbon sequestering in their forest every five years. These checks are to be completed by following a decision making chart which was created looking at census data that included your communities, reflecting your economic and environmental preferences. Social behavior is also accounted for by determining the community activity on the land. Land the community is actively on will be preserved as much as possible. Based on how the forest is responding to the current harvesting and planting methods and how the community uses the land, adjustments are suggested to be implemented for the next five years.

On top of these adjustments, there are planting requirements for the forest manager that are related to how much of the forest is harvested every year. This will ensure that trees continue to be grown and carbon sequestered. There will not be barren land left behind where the forest once was. Instead, we ask the forest manager to harvest a proportion of young, middle aged, and old trees so that there is a variety of trees left in the forest. Our decision chart also introduces a lower bound so that the forest will never be clear cut.

Requiring harvesting to occur across different aged trees results in a steady growth in stored carbon. Large old trees store more carbon annually than young trees. However, harvesting only young trees results in a forest where the population of old trees does not grow. Thus, a proportion harvested from each age class allows us to continue growing and storing lots of carbon, while meeting economic demands for wood products, and maintaining a diverse forest.

Overall, we have a versatile model that creates a forest management plan that will preserve the forest while providing an economically stable harvesting rate. This will provide the jobs and the wood products required to have a blossoming economy in your community and it will provide growing levels of total carbon storage. We are here to better the forest management in your community.