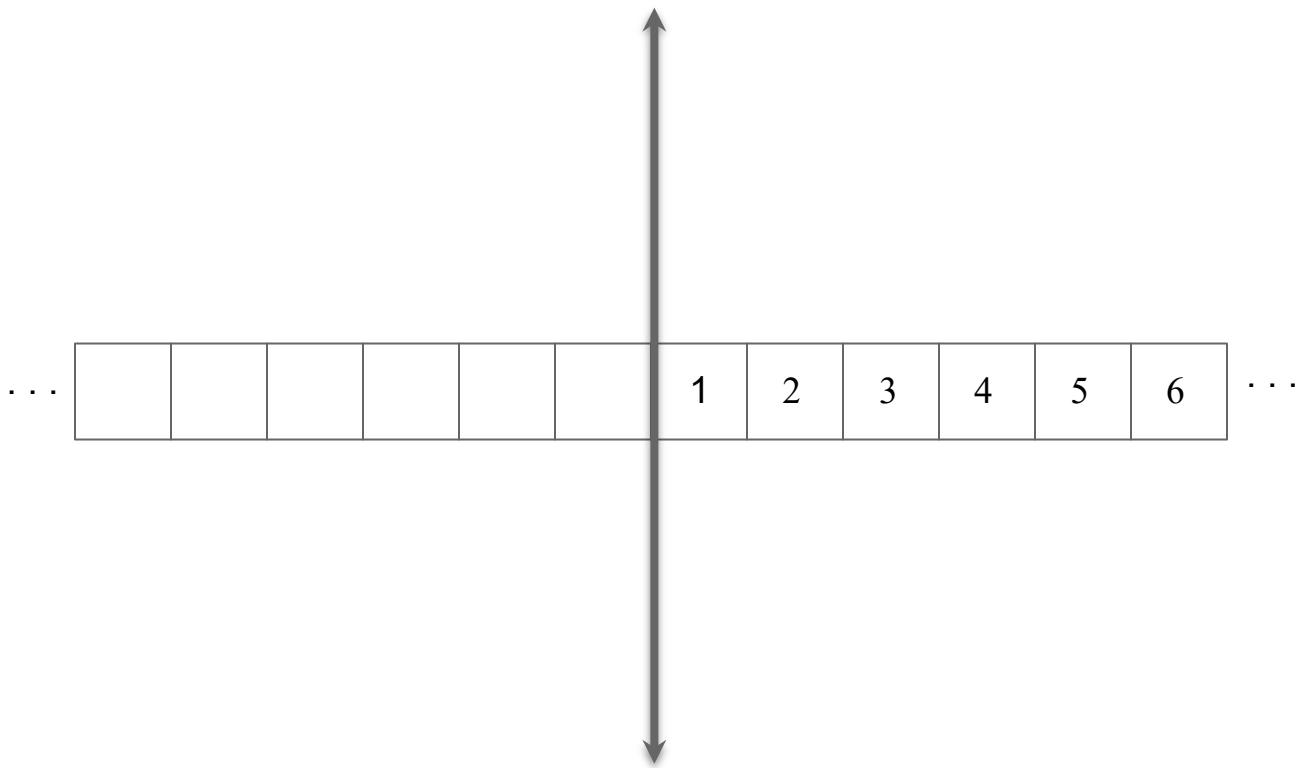# One-dimensional Port-and-Sweep Solitaire Armies

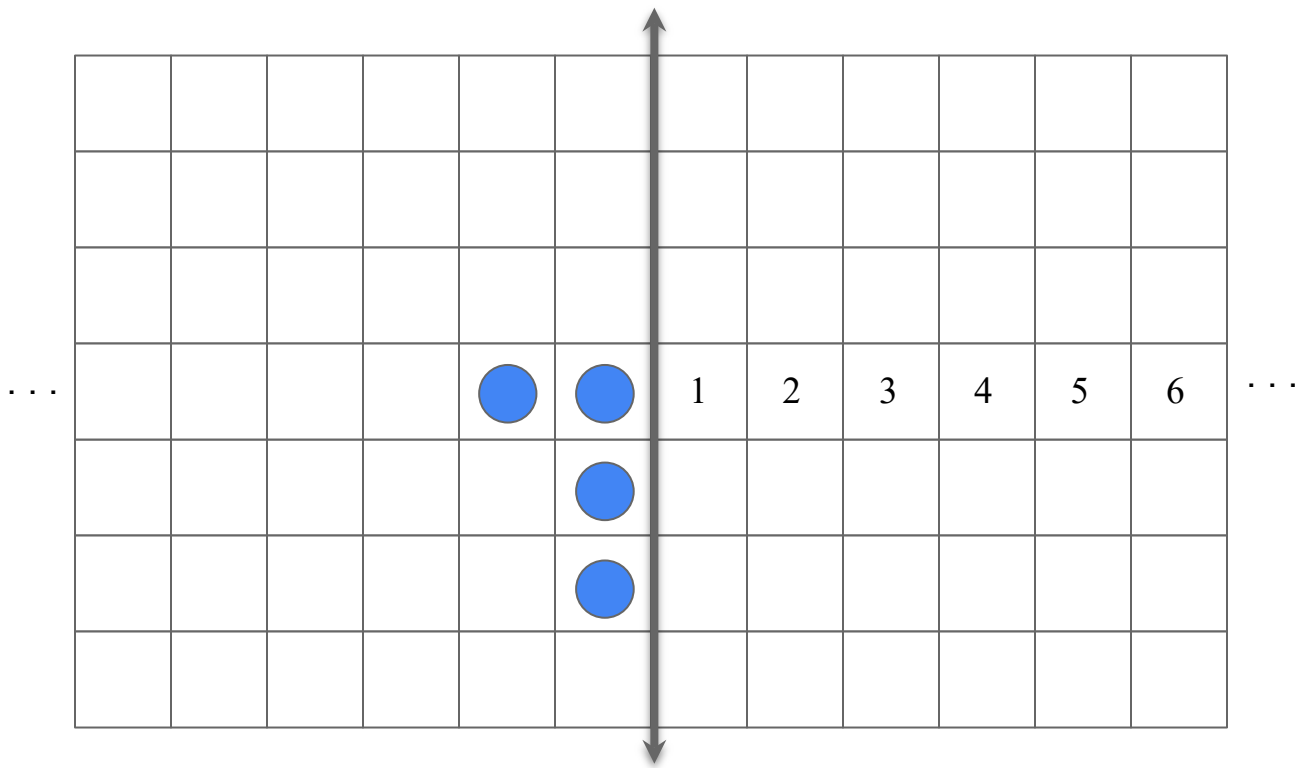Filip Belik and Ha Le

Peg Solitaire

- Goal of reducing to one peg
- Use of jump moves where jumped peg is removed
- Lots of research done on this puzzle alone
  - Conway Pagoda Function
  - NP-Completeness
  - Regular Languages

Solitaire Army Problem 1D

Solitaire Army Problem 2D
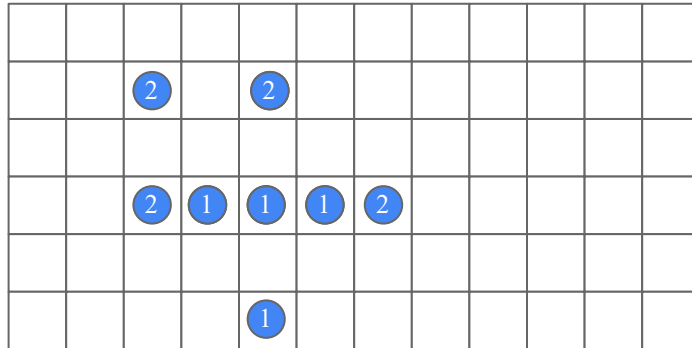
Port -and-Sweep Solitaire (PaSS) Rules

- ⬤ Game is played on a 2D grid similar to Peg Solitaire
- ● Spaces can now hold up to 2 counters
- ● Two types of moves (can be played up, down, left, right).
- ● Port Move   -------- | -2 | 0 | +1 |
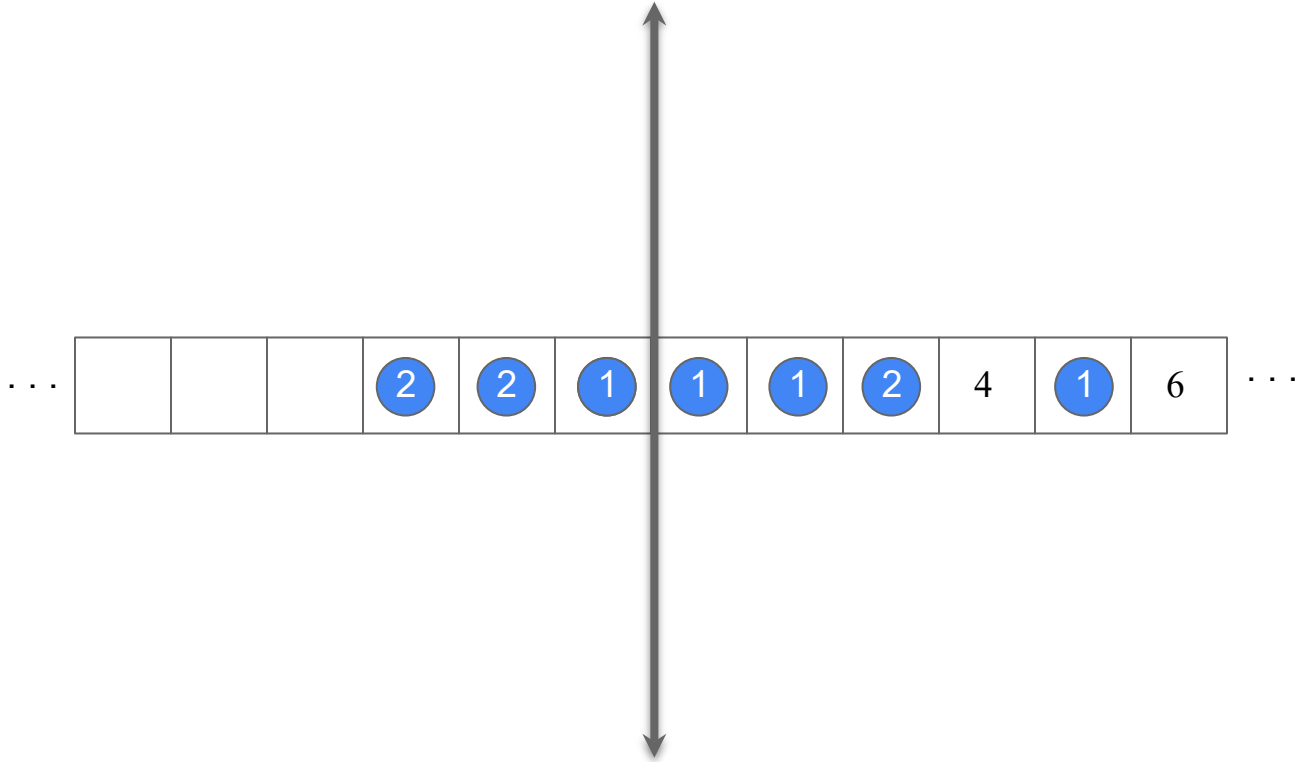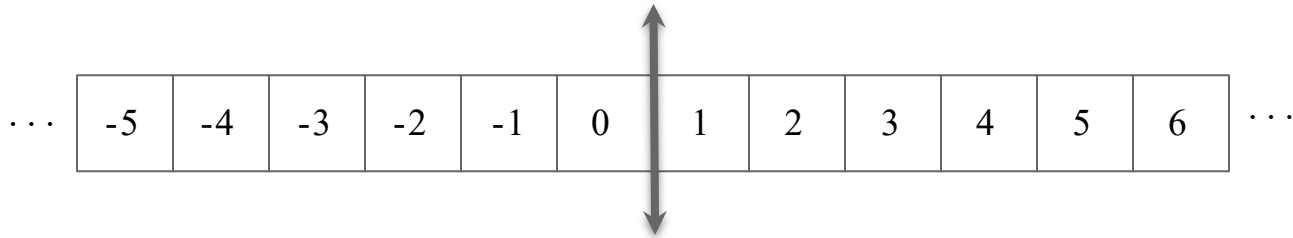- ● Sweep Move   ---- | -1 | -1 | -1 | +2 |

PaSS Army Problem 1D

Q: What is the furthest distance any army can advance?

Resource Counts

- Using the idea of a weight function, α, which gives a specific value to any board configuration

...	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	...

- Let α ≈ 1.2338, the value of a counter at position i is given by α^i
- The *alpha-value* of a board is given by the sum of values of all counters
- Think of the *alpha-value* as a "resource count"
  - Non-increasing value by construction

Resource Counts

- Alpha value of the below board is given as follows
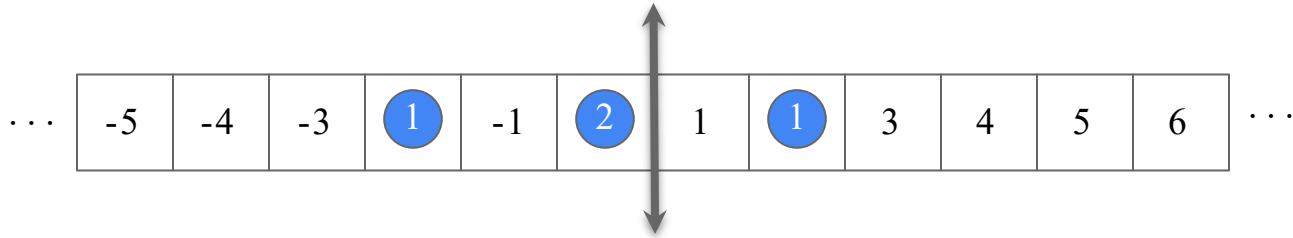
- Alpha value $= \alpha^{-2} + 2\alpha^0 + \alpha^2 \approx 4.18$
- We can calculate the maximum possible initial resource of an army as so
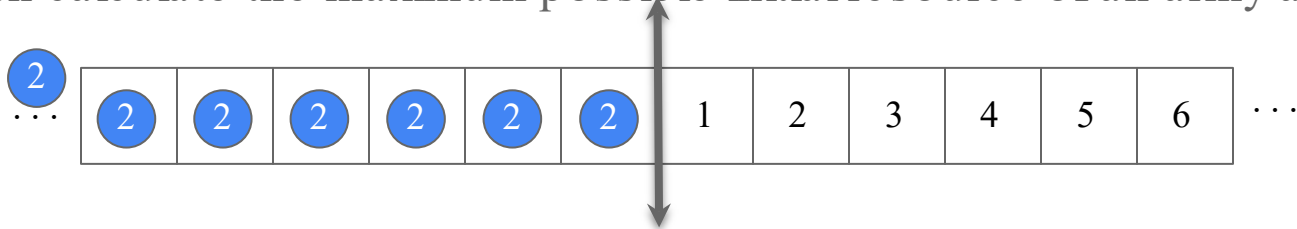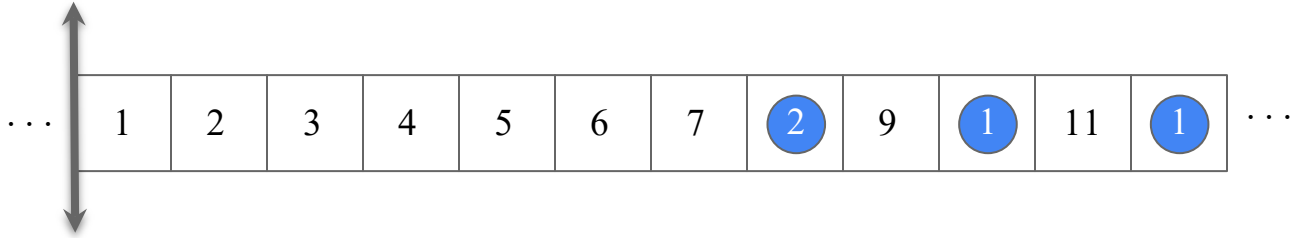

- Maximum army alpha value $= 2\alpha^0 + 2\alpha^{-1} + 2\alpha^{-2} + \ldots \approx 10.56$

Distances of 10+

- Distance of 12 requires too high of a resource count
  - Starting *alpha-value* <= 10.56 <= 12.44 ≈ α^{12}
  - Hence, army advances of 12+ are impossible
- A single counter at cell 10 implies a port from cell 8
  - Starting *alpha-value* <= 10.56 <= 10.74 ≈ $2\alpha^8$
  - Hence, army advances of 10+ are impossible as well

Distance of 9

● We have shown that in order to get a single counter at cell 9, these following moves have to be made at some points:
  ○ A rightwards port from 7 to 9
  ○ A rightwards port from 4 to 6
  ○ A rightwards port from 3 to 5

$$[\text{maximum starting resource}] - [\text{resource loss by moves}] \approx 6.47$$

● However, $\alpha^9 \approx 6.62$
● Hence, army advances of 9 are impossible

Distance of 8

- A distance of 8 is difficult to prove by deductive strategy.
- Assumption:
    - No leftwards (backwards) moves are made
    - No debris left behind
- Approach:
    - Linear algebra (linear combination of vectors) | -2 | 0 | +1 | | -1 | -1 | -1 | +2 |
    - Computer generation
- Able to show given above assumptions, no army can advance a distance of 8

Possible Advances

● Here is the configuration to achieve a distance of 6 with 10 counters
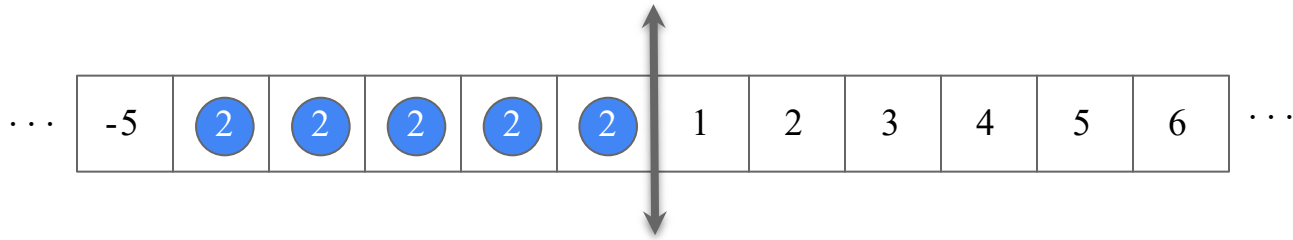
● How many counters do you think are needed to advance 7 spaces?
  ○ 12?
  ○ 15?
  ○ 20?
  ○ 50?

Thank you

- Professor Jacob Siehler
- Stephen Hilding Fund

More reading about game here:

Collection of puzzles to try: